Fairness - Homework

October 26, 2023

1 Simpson's paradox

Table 2. Admissions data for the graduate programs in the six largest majors at University of California, Berkeley.

	Men		*Women*	
Major	*Number of applicants*	*Percent admitted*	*Number of applicants*	*Percent admitted*
A	825	62	108	82
B	560	63	25	68
C	325	37	593	34
D	417	33	375	35
E	191	28	393	24
F	373	6	341	7

Note: University policy does not allow these majors to be identified by name.
Source: The Graduate Division, University of California, Berkeley.

1. What is the percentages of women/men admitted ? Does it seem biased?
2. Compare this result with the detailed percentages by department. Does it seem biased? How can you explain this result?

Size	Treatment A	Treatment B
Small	(96%) 84/87	(87%) 234/270
Large	(73%) 192/263	(68%) 55/80

2 Some fairness criteria

In the binary classification setting, let Y be the target variable, A the sensitive attribute and R the classifier. Recall that the triple (R, A, Y) satisfies the *separation criteria* if $R \perp A \mid Y$ and the *sufficiency criteria* if $Y \perp A \mid R$.

1. Assume that R is a binary classifier and that there are only two groups. What does the separation criteria mean in terms of false positive rate and false negative rate for the two groups, a and b?

2. If we observe the two group-level ROC curves, graphically, which point corresponds to the classifier satisfying the separation criteria?

3. We say that R satisfies *calibration by group* if for all r in the support of R:

$$\mathbb{P}(Y = 1 \mid R = r, A = a) = \mathbb{P}(Y = 1 \mid R = r, A = b) = r$$

   for all r. Show that: if R satisfies sufficiency, there exists a function ℓ such that $\ell(R)$ satisfies calibration by group.

4. Show that: if A is not independent of Y and R is not independent of Y, then independence and separation cannot both hold.